TO‘RTINCHI TARTIBLI MURAKKAB TURDAGI LAPLAS OPERATORI QATNASHGAN TENGLAMA UCHUN CHEGARAVIY MASALA

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Annotatsiya: Maqolada to‘rtinchi tartibli murakkab turdagi tenglamalar uchun chegaraviy masala qaralgan bo‘lib, masalaning yechimining yagonaligi integral energiya usulida, yechimning mavjudligini isbotlashda integral tenglamalari nazariyasidan foydalanilgan.

Kalit so‘zlar: Grin formulasi, yuqori tartibli tenglama, Laplas operatori, yechimning yagonaligi, yechimning mavjudligi, energiya integrali.

КРАЕВАЯ ЗАДАЧА ДЛЯ УРАВНЕНИЯ ЧЕТВЁРТОГО ПОРЯДКА СЛОЖНОГО ТИПА, СОДЕРЖАЩЕГО ОПЕРАТОР ЛАПЛАСА

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Аннотация: В статье рассматривается краевая задача для уравнения четвёртого порядка сложного типа. Единственность решения задачи доказана методом интегральной энергии, а при доказательстве существования решения использованы методы теории интегральных уравнений.

Ключевые слова: формула Грина, уравнение высокого порядка, оператор Лапласа, единственность решения, существование решения, интеграл энергии.

BOUNDARY VALUE PROBLEM FOR AN EQUATION INVOLVING A FOURTH-ORDER LAPLACE OPERATOR OF COMPLEX TYPE

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Abstract: The article considers a boundary value problem for a fourth-order equation of complex type. The uniqueness of the solution is proved by the integral energy method, while the existence of the solution is established using the theory of integral equations.

Keywords: Green’s formula, higher-order equation, Laplace operator, uniqueness of the solution, existence of the solution, energy integral.

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KIRISH

Murakkab turdagi tenglamalar nazariyasi bo‘yicha dastlabki natijalar M.A.Abduraxmonov, S.A.Aldashev, V.F.Volkodavov, V.N.Vragov, T.D.Djo‘rayev, T.Sh.Kalmenov, M.M.Meredov, A.M.Naxushev, M.S.Salohiddinov[1,2], M.M.Smirnov, K.V.Sobitov, M.A.Sadibekov, A.Q.O‘rinov, V.Islomov va boshqalar tomonidan turli yo‘nalishlarda ishlab chiqilgan. Hozirgi kunda yuqori tartibli tenglamalar bilan bir qator olimlar ilmiy izlanishlar olib bormoqda.

Murakkab turdagi tenglamalar uchun chegaraviy masalalar birinchi marta

M.S.Salohiddinov[1,2], T.D.Djo‘rayev[3] tomonidan o‘rganildi. M.S.Salohiddinov va T.D.Djo‘rayev monografiyalarida kompozit va murakkab turdagi tenglamalarni o‘rganishga bag‘ishlangan asarlarning umumiy ko‘rinishi mavjud.

TADQIQOT MATERIALLARI VA METODOLOGIYASI

Ushbu

$$\frac{\partial^2}{\partial y^2}\Delta u + cu = f(x,y) \qquad (1)$$

tenglamani $D=\{(x,y):0<x<1,0<y<1\}$ to‘g‘ri to‘rtburchak sohada qaraymiz. Bu sohaning uchlarini $A(0;0)$, $B(1;0)$, $C(1;1)$, $D(0;1)$ deb belgilaymiz.

Bunda $\Delta \equiv \frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$ - Laplas operatori, $c(x,y)$, $f(x,y)$ - berilgan funksiyalar.

Masalaning qo‘yilishi. Shunday $u(x,y)\in C^2(\overline{D})\cap C^4(D)$ funksiyani topingki, u D sohada (1) tenglamani va quyidagi (2)-(4) shartlarni qanoatlantirsin:

$$\begin{aligned} &u(0,y)=\varphi_0(y) \qquad u(1,y)=\varphi_1(y), \qquad 0<y<1 \\ &u(x,0)=\psi_0(x) \qquad u(x,1)=\psi_1(x), \qquad 0<x<1 \end{aligned} \qquad (2)$$

$$u(x,-x+1)=\psi_2(x), \qquad 0<x<1 \qquad (3)$$

$$a(x)u_y(x,0)+b(x)u_{yy}(x,0)=\psi_3(x), \qquad 0<x<1 \qquad (4)$$

Bu yerda φ_i, ψ_i $(i=0,3)$ lar berilgan funksiyalar bo‘lib, yechimni ko‘rsatilgan sinfdan bo‘lishini ta’minlovchi kelishuv shartlarini qanoatlantiradi.

TADQIQOT NATIJALARI

Masala yechimining yagonaligi.

Teorema. Agar berilgan masalaning yechimi mavjud bo‘lsa va

$$c(x,y)\geq 0,(x,y)\in D\ ,\ \frac{a(x)}{b(x)}\leq 0$$

shartlar bajarilsa, u holda masalaning bittadan ortiq yechimi mavjud emas.

Isbot. Masala yechimining yagonaligini isbotlash uchun (1) - (4) masalaning yechimi 2 ta deb faraz qilamiz. u_1 va u_2 funksiyalar (1)-(4) masalaning yechimi bo‘lsin. U holda

$$u=u_1-u_2$$

ham (1) tenglamani qanoatlantiradi. Bu yechimga **mos chegaraviy shartlar bir jinsli bo‘lib qoladi. Ya’ni**

$$\varphi_i \equiv \psi_i \equiv f(x,y) \equiv 0, \qquad i = 0,1,2,3 \tag{5}$$

Ushbu D_ε soha $D_\varepsilon = \{(x,y): \varepsilon < x < 1-\varepsilon, \varepsilon < y < 1-\varepsilon\}$ ko‘rinishida bo‘lsin. Γ_ε - D_ε sohaning chegarasi.

Berilgan (1) tenglamani $u(x,y)$ ga ko‘paytirib, D_ε soha bo‘yicha integrallaymiz:

$$\iint_{D_\varepsilon} u\left(\frac{\partial^2}{\partial y^2}\Delta u + cu\right)dxdy = 0 \tag{6}$$

D_ε sohada quyidagi tenglik o‘rinli:

$$u\frac{\partial^2}{\partial y^2}\Delta u = u\frac{\partial}{\partial y}\Delta u_y = \frac{\partial}{\partial y}(u\Delta u_y) - \left[\frac{\partial}{\partial x}(u_y u_{xy}) - u_{xy}^2 + \frac{\partial}{\partial y}(u_y u_{yy}) - u_{yy}^2\right] \tag{7}$$

(7) tenglikni (6) ga qo‘yamiz va Grin formulasini qo‘llaymiz:

$$\int_{\Gamma_\varepsilon} u\Delta u_y dx - \int_{\Gamma_\varepsilon} u_y u_{xy} dy - \int_{\Gamma_\varepsilon} u_y u_{yy} dx + \iint_{D_\varepsilon} (u_{xy}^2 + u_{yy}^2 + c(x,y)u^2)dxdy = 0 \tag{8}$$

Bu yerda (5) ni inobatga olib, (2)-(4) chegaraviy shartlardan foydalanamiz va har bir integralni alohida hisoblaymiz:

$$J_1 = \int_{\Gamma_\varepsilon} u\Delta u_y dx$$

$\varepsilon \to 0$ da bir jinsli shartlarga asosan $J_1 = 0$ tenglik o‘rinli.

(8) tenglikning 2-hadidagi integralda AB va CD da $dy = 0$ bo‘ladi. U holda quyidagi

$$J_2 = \int_{\Gamma_\varepsilon} u_y u_{xy} dy = \int_{BC} u_y u_{xy}(1,y)dy + \int_{DA} u_y u_{xy}(0,y)dy$$

tenglik o‘rinli. Bir jinsli shartga asosan $u_y = 0$ bo‘lib, $J_2 = \int_{\Gamma_\varepsilon} u_y u_{xy} dy = 0$ ekanligi kelib chiqadi.

BC va DA chiziqlarda $u_y u_{yy}$ dan olingan integralni qarasak, BC va DA da $dx = 0$. Bundan

$$J_3 = \int_{\Gamma_\varepsilon} u_y u_{yy} dx = \int_{AB} u_y u_{yy}(x,0)dx + \int_{CD} u_y u_{yy}(x,1)dx$$

tenglik o‘rinli. Chegaraviy shartlarni hisobga olib,

$$\int_{AB} u_y u_{yy}(x,0)dx + \int_{CD} u_y u_{yy}(x,1)dx = \int_{CD} u_y u_{yy}(x,1)dx + \int -\frac{a(x)}{b(x)} u_y^2(x,0)dx$$

ko‘rinishga kelamiz. Bu integralda (3) shartdan foydalanamiz, ya’ni $x = -y + 1$ va $dy = -dx$ almashtirish bajaramiz, hamda teorema shartlariga asosan

$$\int_0^1 u_y(x,1)u_{yy}(x,1)dx = 0$$

tenglikka kelamiz. U holda (8) ifoda $\varepsilon \to 0$ da

$$-\int_0^1 \frac{a(x)}{b(x)}u_y^2(x,0)dx + \iint_D (u_{xy}^2 + u_{yy}^2)dxdy + \iint_D c(x,y)u^2 dxdy = 0 \qquad (9)$$

ko‘rinishni oladi.

Oxirgi (9) tenglikda:

a) $c(x,y) > 0$, $\frac{a(x)}{b(x)} \le 0$ bo‘lsa, u holda $u \equiv 0$ ekani kelib chiqadi;

b) $c(x,y) = 0$, $\frac{a(x)}{b(x)} = 0$ bo‘lsa, D sohada $u_{xy} = u_{yy} = const$ bo‘ladi. $u \in C^2(\bar{D})$ ekanligidan, $u_{xy} = u_{yy} = 0$ tenglikni olish mumkin. Ulardan birini $u_{yy} = 0$ tenglamani integrallab, $u = f(x)y + f_1(x)$ yechimga ega bo‘lamiz. Bir jinsli chegaraviy shartlarga asosan $u \equiv 0$ ekani kelib chiqadi.

c) $c(x,y) = 0$, $\frac{a(x)}{b(x)} < 0$ bo‘lsa, $u_y^2(x,0) = 0$ bo‘ladi. Bundan $u \equiv 0$. Demak $u_1 = u_2$ bo‘lib, masala yechimi yagona bo‘ladi. **Teorema isbotlandi.**

Masala yechimining mavjudligi

2-teorema. Agar quyidagi

$$\varphi_i''(y),(0 \le y \le 1),\psi_i''(x), i = 0,1, \quad \psi_3(x),\psi_2(x),\psi_3'(x),\psi_2'(x), f(x,y)$$

funksiyalar $0 \le x \le 1$, $0 \le y \le 1$ oraliqlarda uzluksiz bo‘lib, $c(x,y) \ge 0$, $(x,y) \in D$ $\frac{a(x)}{b(x)} \le 0$ shartlar bajarilsa, masala yechimi mavjud.

Isbot. $u_{yy} = \upsilon$ belgilash yordamida (1) tenglama quyidagi ko‘rinishni oladi:

$$\Delta\upsilon = f(x,y) - c(x,y)u(x,y) \qquad (10)$$

(10) tenglama uchun masalaning (2), (3) va (4) chegaraviy shartlaridan foydalanib quyidagicha shartlarga ega bo‘lamiz:

$$\begin{aligned}
&\upsilon(0,y) = u_{yy}(0,y) = \varphi_0''(y) \\
&\upsilon(x,0) = u_{yy}(x,0) = \frac{\psi_3(x)}{b(x)} - \frac{a(x)}{b(x)}u_y(x,0) = \overline{\psi}_3(x) \\
&\upsilon(1,y) = u_{yy}(1,y) = \varphi_1''(y) \\
&\upsilon(x,1) = u_{yy}(x,1) = \overline{\psi}_1(x)
\end{aligned} \qquad (11)$$

Bu yerda $u_{yy}(x,1) = \overline{\psi_1}(x)$ va $u_y(x,0) = \overline{\psi_3}(x)$ - funksiyalar noma’lum.

(10) tenglamaning (11) chegaraviy shartlarni qanoatlantiruvchi yechimi quyidagicha bo‘ladi[1]:

$$\upsilon(x,y)=\int_0^1 G_\eta(x,y,\xi,0)\overline{\psi_3}(\xi)d\xi-\int_0^1 G_\eta(x,y,\xi,1)\overline{\psi_1}(\xi)d\xi-$$

$$-\int_0^1 G_\xi(x,y,1,\eta)\varphi_1''(\eta)d\eta+\int_0^1 G_\xi(x,y,0,\eta)\varphi_0''(\eta)d\eta-$$

$$-\iint_D G(x,y,\xi,\eta)c(\xi,\eta)u(\xi,\eta)d\xi d\eta+F(x,y) \qquad (12)$$

Bu yerda

$$F(x,y)=\iint_D G(x,y,\xi,\eta)f(\xi,\eta)d\xi d\eta,$$

$$G(x,y,\xi,\eta)=-\ln r+g(x,y,\xi,\eta), \quad \left(r=\sqrt{(x-\xi)^2+(y-\eta)^2}\right)$$

Laplas tenglamasi uchun to‘rtburchak sohada Dirixle masalasining Grin funksiyasi.

Yuqoridagi belgilashga asosan quyidagi 2 ta masalaga ega bo‘lamiz:

$$\begin{cases} w_y=\upsilon(x,y) \\ w(x,0)=\psi_4(x) \end{cases} \qquad (13)$$

va

$$\begin{cases} u_y=\upsilon(x,y) \\ u(x,-x+1)=\psi_2(x) \end{cases} \qquad (14)$$

Bu masalarda $\psi_4(x)$ - noma’lum funksiya. U holda (13) masalaning yechimi:

$$w=\int_0^y \upsilon(x,t)dt+\psi_4(x) \qquad (15)$$

ko‘rinishida bo‘ladi. Bu ifodadan foydalanib, (14) ni yechimini quyidagicha yozamiz:

$$u=\int_{1-x}^{y} w(x,t)dt+\psi_2(x) \qquad (16)$$

Endi (15) ga (12) tenglikni qo‘ysak:

$$w=\int_0^y\left[\int_0^1 G_\eta(x,t,\xi,0)\overline{\psi_3}(\xi)d\xi-\int_0^1 G_\eta(x,t,\xi,1)\overline{\psi_1}(\xi)d\xi-\right.$$

$$\left.-\int_0^1 G_\xi(x,t,1,\eta)\varphi_1''(\eta)d\eta+\int_0^1 G_\xi(x,t,0,\eta)\varphi_0''(\eta)d\eta\right]dt-$$

$$-\int_0^y\iint_D G(x,t,\xi,\eta)c(\xi,\eta)u(\xi,\eta)d\xi d\eta dt+\psi_4(x)+$$

$$+\int_{0}^{y}\left[\iint_{D} G(x,t,\xi,\eta)\, f(\xi,\eta)\, d\xi d\eta\right] dt \qquad (17)$$

ko‘rinishga keladi.

(17) ifodani (16) formulaga qo‘yib, quyidagiga ega bo‘lamiz:

$$u=\int_{1-x}^{y}\left[\int_{0}^{t}\left[\int_{0}^{1} G_{\eta}(x,t_1,\xi,0)\overline{\psi_3}(\xi)d\xi-\int_{0}^{1} G_{\eta}(x,t_1,\xi,1)\overline{\psi_1}(\xi)d\xi-\right.\right.$$

$$\left.\left.-\int_{0}^{1} G_{\xi}(x,t_1,1,\eta)\varphi_1''(\eta)d\eta+\int_{0}^{1} G_{\xi}(x,t_1,0,\eta)\varphi_0''(\eta)d\eta\right]dt_1\right]dt- \qquad (18)$$

$$-\int_{1-x}^{y}\left[\int_{0}^{t}\iint_{D} G(x,t_1,\xi,\eta)c(\xi,\eta)u(\xi,\eta)d\xi d\eta dt_1-\psi_4(x)\right]dt+$$

$$+\int_{1-x}^{y}\int_{0}^{t}\left[\iint_{D} G(x,t_1,\xi,\eta)\, f(\xi,\eta)\, d\xi d\eta\right]dt_1 dt+\psi_2(x)$$

(18) tenglikdagi ba’zi integrallarni hisoblaymiz:

$$J_0=\int_{1-x}^{y}\left(\int_{0}^{t} G(x,t_1,\xi,\eta)dt_1\right)dt=\int_{1-x}^{y}\int_{0}^{t}\left(-\ln\sqrt{(x-\xi)^2+(t_1-\eta)^2}+g(x,t_1,\xi,\eta)\right)dt_1 dt=$$

$$=-\frac{1}{4}\left((x-\xi)^2+(y-\eta)^2\right)\ln\left((x-\xi)^2+(y-\eta)^2\right)+g_1(x,y,\xi,\eta)$$

$$J_1=\int_{1-x}^{y}\int_{0}^{1}\left[\int_{0}^{t} G_{\eta}(x,t_1,\xi,1)\overline{\psi_1}(\xi)d\xi\right]dt_1 dt=\int_{1-x}^{y}\left[\int_{0}^{1}\overline{\psi_1}(\xi)d\xi\int_{0}^{t} G_{\eta}(x,t_1,\xi,1)dt_1\right]dt=$$

$$=\int_{0}^{1}\overline{\psi_1}(\xi)\left[(y-1)\ln\left[\frac{(x-\xi)^2+(y-1)^2}{(x-\xi)^2+1}\right]+x\ln\left[\frac{(x-\xi)^2+x^2}{(x-\xi)^2+1}\right]+g_2(x,y,\xi,1)\right]d\xi$$

$$J_2=\int_{1-x}^{y}\int_{0}^{1}\left[\int_{0}^{t} G_{\eta}(x,t_1,\xi,0)\overline{\psi_3}(\xi)d\xi\right]dt_1 dt=\int_{1-x}^{y}\left[\int_{0}^{1}\overline{\psi_3}(\xi)d\xi\int_{0}^{t} G_{\eta}(x,t_1,\xi,0)dt_1\right]dt=$$

$$=\int_{0}^{1}\overline{\psi_3}(\xi)\left[y\ln\left[\frac{(x-\xi)^2+y^2}{(x-\xi)^2}\right]+(1-x)\ln\left[\frac{(x-\xi)^2+(1-x)^2}{(x-\xi)^2}\right]+g_3(x,y,\xi,0)\right]d\xi$$

bu yerda

$$g_1=-\frac{1}{2}\int_{1-x}^{y}\left[\eta\ln\left((x-\xi)^2+\eta^2\right)-2(t-\eta)-2\eta+2arctg\frac{t-\eta}{x-\xi}-2arctg\frac{-\eta}{x-\xi}\right]dt+$$

$$+\int_{1-x}^{y}\int_{0}^{t}g(x,t_1,\xi,\eta)dt_1dt$$

$$g_2=-2(y-x-1)+2(x-\xi)\left(arctg\frac{y-1}{x-\xi}-arctg\frac{-x}{x-\xi}\right)+\int_{1-x}^{y}\int_{0}^{t}g_\eta(x,t_1,\xi,1)dt_1dt$$

$$g_3=-2(y+x-1)+2(x-\xi)\left(arctg\frac{y}{x-\xi}-arctg\frac{1-x}{x-\xi}\right)+\int_{1-x}^{y}\int_{0}^{t}g_\eta(x,t_1,\xi,0)dt_1dt$$

Topilganlardan foydalanib, $u(x,y)$ ni soddalashtiramiz:

$$u(x,y)+\frac{1}{4}\iint_D R(x,y,\xi,\eta)u(\xi,\eta)d\xi d\eta=\overline{F}(x,y) \qquad (19)$$

Bu yerda

$$\overline{F}(x,y)=F_1(x,y)+F_2(x,y)$$

$$F_1(x,y)=\frac{1}{4}\iint_D \overline{R}(x,y,\xi,\eta)f(\xi,\eta)d\xi d\eta$$

$$F_2(x,y)=\int_0^1\left[\int_{1-x}^{y}\int_0^t G_\xi(x,t_1,0,\eta)dt_1dt\right]\varphi_0''(\eta)d\eta-\int_0^1\left[\int_{1-x}^{y}\int_0^t G_\xi(x,t_1,1,\eta)dt_1dt\right]\varphi_1''(\eta)d\eta-$$

$$-\int_0^1\overline{\psi_1}(\xi)\left[(y-1)\ln\left[\frac{(x-\xi)^2+(y-1)^2}{(x-\xi)^2+1}\right]+x\ln\left[\frac{(x-\xi)^2+x^2}{(x-\xi)^2+1}\right]+g_2(x,y,\xi,1)\right]d\xi-$$

$$-\int_0^1\overline{\psi_3}(\xi)\left[y\ln\left[\frac{(x-\xi)^2+y^2}{(x-\xi)^2}\right]+(1-x)\ln\left[\frac{(x-\xi)^2+(1-x)^2}{(x-\xi)^2}\right]+g_3(x,y,\xi,0)\right]d\xi+$$

$$+(y-1+x)\psi_4(x)+\psi_2(x);$$

$$R(x,y,\xi,\eta)=\overline{R}(x,y,\xi,\eta)c(\xi,\eta)$$

$$\overline{R}(x,y,\xi,\eta)=\left((x-\xi)^2+(y-\eta)^2\right)\ln\left((x-\xi)^2+(y-\eta)^2\right)+g_1(x,y,\xi,\eta)$$

(19) – Fredgolmning 2-tur integral tenglamasi bo‘lib, ko‘rish mumkinki, uning yadrosi $R(x,y,\xi,\eta)$ - uzluksiz funksiya, $\overline{F}(x,y)$ funksiya ham chegaraviy shartlardagi berilgan

funksiyalarga qo‘yilgan shartlarga asosan uzluksiz bo‘ladi. Shuning uchun, unga Fredgolm teoremalarini qo‘llash mumkin. Shunday qilib, uning yechimi[1]:

$$u(x,y)=\overline{F}(x,y)-\frac{1}{4}\iint\limits_{D}\Gamma(x,y,\xi,\eta)\overline{F}(\xi,\eta)d\xi d\eta \tag{20}$$

ko‘rinishida bo‘ladi. Bu yerda $\Gamma(x,y,\xi,\eta)-R(x,y,\xi,\eta)$ yadroning rezolventasi.

$\overline{F}(x,y)$ ni ifodasidan foydalanib, (20) ni quyidagicha yozamiz:

$$\begin{aligned}
u(x,y)=&-\int\limits_0^1\overline{\psi_1}(\xi)\left[(y-1)\ln\left[\frac{(x-\xi)^2+(y-1)^2}{(x-\xi)^2+1}\right]+\right.\\
&\left.+x\ln\left[\frac{(x-\xi)^2+x^2}{(x-\xi)^2+1}\right]+g_2(x,y,\xi,1)\right]d\xi+\\
&+\int\limits_0^1\overline{\psi_3}(\xi)\left[y\ln\left[\frac{(x-\xi)^2+y^2}{(x-\xi)^2}\right]+(1-x)\ln\left[\frac{(x-\xi)^2+(1-x)^2}{(x-\xi)^2}\right]+g_3(x,y,\xi,0)\right]d\xi-\\
&-\iint\limits_D\Gamma(x,y,\xi,\eta)\left[\int\limits_0^1\overline{\psi_1}(\xi_1)\left[(\eta-1)\ln\left[\frac{(\xi-\xi_1)^2+(\eta-1)^2}{(x-\xi_1)^2+1}\right]+\right.\right.\\
&\left.\left.+\xi\ln\left[\frac{(\xi-\xi_1)^2+\xi^2}{(\xi-\xi_1)^2+1}\right]+g_2(\xi,\eta,\xi_1,1)\right]\right]d\xi_1 d\xi d\eta+\\
&+\iint\limits_D\Gamma(x,y,\xi,\eta)\left[\int\limits_0^1\overline{\psi_3}(\xi_1)\left[\eta\ln\left[\frac{(\xi-\xi_1)^2+\eta^2}{(\xi-\xi_1)^2}\right]+\right.\right.\\
&\left.\left.+(1-\xi)\ln\left[\frac{(\xi-\xi_1)^2+(1-\xi)^2}{(\xi-\xi_1)^2}\right]+g_3(\xi,\eta,\xi_1,0)\right]\right]d\xi_1 d\xi d\eta+\\
&+(y-1+x)\psi_4(x)+F_3(x,y)
\end{aligned} \tag{21}$$

Bu yerda

$$F_3(x,y)=\frac{1}{4}\iint\limits_D R(x,y,\xi,\eta)f(\xi,\eta)d\xi d\eta+\int\limits_0^1\left[\int\limits_{1-x}^{y}\int\limits_0^t G_\xi(x,t_1,0,\eta)dt_1dt\right]\varphi_0''(\eta)d\eta-$$

$$-\int\limits_0^1\left[\int\limits_{1-x}^{y}\int\limits_0^t G_\xi(x,t_1,1,\eta)dt_1dt\right]\varphi_1''(\eta)d\eta+\psi_2(x)+\iint\limits_D\Gamma(x,y,\xi,\eta)(\eta-1+\xi)\psi_4(\xi)d\xi d\eta$$

$u_{yy}(x,1)=\overline{\psi_1}(x)$ ekanligidan foydalanib, (21) da y bo‘yicha ikki marta hosila olamiz va $y=1$ qo‘ysak,

$$\overline{\psi_1}(x)=\int_0^1 R_1(x,\xi_1)\overline{\psi_1}(\xi_1)d\xi_1+\int_0^1 k_1(x,\xi_1)\overline{\psi_3}(\xi_1)d\xi_1+F_3(x) \tag{22}$$

ko‘rinishidagi tenglamaga kelamiz. Bu yerda

$$R_1(x,\xi)=\iint_D \Gamma_{yy}(x,1,\xi_1,1)\left[(1-\eta)\ln\left[\frac{(\xi-\xi_1)^2+(\eta-1)^2}{(x-\xi_1)^2+1}\right]-\right.$$

$$\left.-\xi\ln\left[\frac{(\xi-\xi_1)^2+\xi^2}{(\xi-\xi_1)^2+1}\right]-g_2(\xi,\eta,\xi_1,1)\right]d\xi d\eta+g_{2yy}(x,1,\xi,1)$$

$$k_1(x,\xi_1)=\frac{2}{(x-\xi_1)^2+1}+\frac{4(x-\xi_1)^2}{\left((x-\xi_1)^2+1\right)^2}+\iint_D \Gamma_{yy}(x,1,\xi,\eta)\left[\eta\ln\left[\frac{(\xi-\xi_1)^2+\eta^2}{(\xi-\xi_1)^2}\right]+\right.$$

$$\left.+(1-\xi)\ln\left[\frac{(\xi-\xi_1)^2+(1-\xi)^2}{(\xi-\xi_1)^2}\right]+g_3(\xi,\eta,\xi_1,0)\right]d\xi d\eta+g_{3yy}(x,1,\xi,0)$$

$$F_3(x)=F_{3yy}(x,1)$$

Endi $u_{yy}(x,0)=\overline{\psi_3}(x)$ ekanligidan (21) dan y bo‘yicha ikki marta hosila olib, $y=0$ ni o‘rniga qo‘yib,

$$\overline{\psi_3}(x)=\int_0^1 R_2(x,\xi_1)\overline{\psi_3}(\xi_1)d\xi_1+\int_0^1 k_2(x,\xi_1)\overline{\psi_1}(\xi_1)d\xi_1+\overline{F_3}(x) \tag{23}$$

tenglamaga kelamiz. Bu yerda

$$R_2(x,\xi_1)=\iint_D \Gamma_{yy}(x,0,\xi,\eta)\left[\eta\ln\left[\frac{(\xi-\xi_1)^2+\eta^2}{(\xi-\xi_1)^2}\right]+\right.$$

$$\left.+(1-\xi)\ln\left[\frac{(\xi-\xi_1)^2+(1-\xi)^2}{(\xi-\xi_1)^2}\right]+g_3(\xi,\eta,\xi_1,0)\right]d\xi d\eta+g_{3yy}(x,0,\xi,0)$$

$$k_2(x,\xi_1)=\frac{2\left((x-\xi_1)^2+1\right)+4(x-\xi_1)^2}{\left((x-\xi_1)^2+1\right)^2}-\iint_D \Gamma_{y}(x,1,\xi,\eta)\left[(\eta-1)\ln\left[\frac{(\xi-\xi_1)^2+(\eta-1)^2}{(x-\xi_1)^2+1}\right.\right.$$

$$\left.+\xi\ln\left[\frac{(\xi-\xi_1)^2+\xi^2}{(\xi-\xi_1)^2+1}\right]+g_2(\xi,\eta,\xi_1,1)\right]d\xi d\eta+g_{2yy}(x,0,\xi,1)$$

$$\overline{F_3}(x)=F_{3yy}(x,0)$$

Shunday qilib, noma'lum funksiyalarga nisbatan Fredgolmning 2-tur integral tenglamalarini hosil qilamiz. Berilgan funksiyalarga qo'yilgan shartlarga asosan (22) va (23) tenglamalarni o'ng tomoni uzluksiz funksiyalar bo'lib, $x=0$, $x=1$ nuqtalarda chegaralangan bo'ladi:

$$\begin{cases}\overline{\psi_1}(x)=\int_0^1 R_1(x,\xi_1)\overline{\psi_1}(\xi_1)d\xi_1+\int_0^1 k_1(x,\xi_1)\overline{\psi_3}(\xi_1)d\xi_1+F_3(x)\\ \overline{\psi_3}(x)=\int_0^1 R_2(x,\xi_1)\overline{\psi_3}(\xi_1)d\xi_1+\int_0^1 k_2(x,\xi_1)\overline{\psi_1}(\xi_1)d\xi_1+\overline{F_3}(x)\end{cases}$$

Bu sistemani yechib noma'lumlarni topamiz[5]:

$$\overline{\psi_1}(x)=F_3(x)+\int_0^1\Gamma_1(x,\xi_1)F_3(\xi_1)d\xi_1+\int_0^1\Gamma_1(x,\xi_1)\overline{F_3}(\xi_1)d\xi_1,$$

$$\overline{\psi_3}(x)=\overline{F_3}(x)+\int_0^1\Gamma_2(x,\xi_1)\overline{F_3}(\xi_1)d\xi_1+\int_0^1\Gamma_2(x,\xi_1)F_3(\xi_1)d\xi_1.$$

Bu yerda $\Gamma_1(x,\xi_1)$ va $\Gamma_2(x,\xi_1)$ lar mos holda $R_1(x,\xi_1)$ va $R_2(x,\xi_1)$ yadrolar rezolventasi.

$\psi_4(x)$ ni topish uchun (11) dagi

$$\frac{\psi_3(x)}{b(x)}-\frac{a(x)}{b(x)}u_y(x,0)=\overline{\psi}_3(x)$$

belgilashdan foydalanamiz:

$$\psi_4(x)=\frac{\psi_3(x)}{a(x)}-\frac{b(x)}{a(x)}\overline{F_3}(x)-\frac{b(x)}{a(x)}\int_0^1\Gamma_2(x,\xi_1)\overline{F_3}(\xi_1)d\xi_1-\frac{b(x)}{a(x)}\int_0^1\Gamma_2(x,\xi_1)F_3(\xi_1)d\xi_1$$

Bu tenglikda ba'zi almashtirishlarni bajarsak, quyidagi

$$\psi_4(x)=\frac{b(x)}{a(x)}\int_0^1 R_3(x,\xi)\psi_4(\xi)d\xi+F_5(x) \tag{24}$$

integral tenglamani hosil qilamiz. Bu yerda

$$R_3(x,\xi)=\int_0^1\Gamma(x,1,\xi,\eta)(1-\xi-\eta)d\eta$$

$$F_5(x,\xi)=\frac{\psi_3(x)}{a(x)}-\frac{b(x)}{a(x)}\overline{F_3}(x)-\frac{b(x)}{a(x)}\int_0^1\Gamma_2(x,\xi_1)\overline{F_3}(\xi_1)d\xi_1-\frac{b(x)}{a(x)}\int_0^1\Gamma_2(x,\xi_1)F_3(\xi_1)d\xi_1+$$

$$+\iint_D\Gamma(x,y,\xi,\eta)(\eta-1+\xi)\psi_4(\xi)d\xi d\eta$$

(24) tenglikda ham teorema shartlariga asosan o‘ng tomoni uzluksiz funksiyalar. Tenglamaning yechimi mavjud. Bundan (24) yordamida $\psi_4(x)$ ni aniqlab, $\overline{\psi_1}(x)$, $\overline{\psi_3}(x)$ va $\psi_4(x)$ larni (21) ifodaga olib borib, berilgan masala yechimini topamiz.

MUHOKAMA

Olingan natijalar (1) tenglama uchun qo‘yilgan chegaraviy masalaning to‘g‘ri qo‘yilganligini ko‘rsatadi. Yechimning yagonaligini isbotlashda qo‘llanilgan integral energiya usuli tenglamani mos ko‘paytuvchiga ko‘paytirish va Grin formulasini qo‘llash orqali (8) ko‘rinishdagi energetik tenglikni hosil qilish imkonini berdi. Teorema shartlarida koeffitsiyentlarga qo‘yilgan cheklovlar aynan shu tenglikdagi hadlarning ishorasini aniqlaydi va yagonalikni ta’minlovchi hal qiluvchi omil bo‘lib xizmat qiladi.

Yechimning mavjudligini isbotlashda esa masala Laplas tenglamasi uchun to‘g‘ri to‘rtburchak sohada Dirixle masalasining Grin funksiyasi yordamida integral ko‘rinishga keltirildi. Natijada noma’lum funksiyalarga nisbatan Fredgolmning ikkinchi tur integral tenglamalari — (19), (22), (23) va (24) — hosil qilindi. Ushbu tenglamalar yadrolarining uzluksizligi hamda o‘ng tomonlarining chegaralanganligi Fredgolm teoremalarini qo‘llash imkonini beradi, bu esa yechimning mavjudligini kafolatlaydi.

Mazkur ish M.S.Salohiddinov [1,2] va T.D.Djo‘rayev [3] tomonidan kompozit va murakkab turdagi tenglamalar uchun ishlab chiqilgan yondashuvlarni to‘rtinchi tartibli tenglamalar sinfiga kengaytiradi. O.S.Zikirov va K.S.Gaziyev [4] tomonidan uchinchi tartibli murakkab turdagi tenglama uchun olingan natijalardan farqli o‘laroq, bu yerda tenglama tarkibida Laplas operatorining qatnashishi Dirixle masalasining Grin funksiyasidan bevosita foydalanish imkonini beradi va integral tenglamalar nazariyasini qo‘llash yo‘lini soddalashtiradi.

XULOSA

Xulosa qilib aytsak, qo‘yilgan masala yechimi yagonaligi integral energiya usuli yordamida, masala yechimi mavjudligini ifodalovchi teorema integral tenglamalar nazariyasi yordamida tavsiflandi. Olingan natijalar yuqori tartibli elliptik operator qatnashgan tenglamalar nazariyasini rivojlantirishda katta yordam beradi.

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